

Unpolarized quasielectrons and the spin polarization at filling fractions between $\nu = 1/3$ and $\nu = 2/5$

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Abstract

We prove that for a hard core interaction the ground state spin polarization in the low Zeeman energy limit is given by $P = 2/\nu - 5$ for filling fractions in the range $1/3 \leq \nu \leq 2/5$. The same result holds for a Coulomb potential except for marginally small magnetic fields. At the magnetic fields $B < 20T$ unpolarized quasielectrons can manifest themselves by a characteristic peak in the I-V characteristics for tunneling between two $\nu = 1/3$ ferromagnets.

I. INTRODUCTION

Recent results [1] on spin depolarization at quantum Hall systems in the vicinity of filling fraction 1 provoke questions about the spin polarization at other filling fractions. A potentially interesting range of filling fractions is the interval $8/5 \leq \nu \leq 5/3$, which is equivalent to $1/3 \leq \nu \leq 2/5$ by particle-hole symmetry $\nu \leftrightarrow 2 - \nu$. The $\nu = 1/3$ state is a ferromagnetic Laughlin state [2], while the $\nu = 2/5$ is a ferromagnet or a spin-singlet depending on the Zeeman energy. For low Zeeman energy the question arises how the spin polarization changes between the limiting ferromagnetic and spin-singlet states. In this paper we give an answer to this question.

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II. SPIN POLARIZATION IN THE RANGE $1/3 < \nu \leq 2/5$ FOR A HARD CORE POTENTIAL

The ferromagnetic $\nu = 1/3$ ground state is described by the celebrated Laughlin wave function

$$\psi_{1/3}(z_k) = \prod_{m>n=1}^N (z_m - z_n)^3 |\downarrow_1 \dots \downarrow_N \rangle . \quad (1)$$

This state is an exact zero energy eigenstate of the hard core potential [3]

$$V(z) = \infty \delta(z) + \lambda \nabla^2 \delta(z) \quad (2)$$

with Haldane's [4] pseudopotentials $V_0 = \infty$, $V_1 > 0$ and $V_k = 0$ for $k \geq 2$. It is the unique zero energy eigenstate for arbitrarily small but nonzero Zeeman energy.

For vanishing Zeeman energy the unique zero energy eigenstate at the filling fraction of $2/5$ is [6] the Halperin's spin-singlet $(3, 3, 2)$ state [5]

$$\psi_{2/5}(z_k) = \prod_{i>j=1}^{N/2} (z_i - z_j)^3 \prod_{k>l=N/2}^N (z_k - z_l)^3 \prod_{m=0}^{N/2} \prod_{n=N/2}^N (z_m - z_n)^2 |\uparrow_1 \dots \uparrow_{N/2} \downarrow_{N/2+1} \dots \downarrow_N \rangle . \quad (3)$$

We display only one spinor component, the one where the spins of the electrons $1, \dots, N/2$ are pointing up and those of the electrons $N/2 + 1, \dots, N$ are pointing down.

$\nu = 1/3$ is the highest filling fraction for which the polarized ground state can still have zero energy [3,6]. For $\nu > 1/3$ electrons are packed together too close, such that some pairs of electrons have to be in a state of relative angular momentum 1. This effect gives rise to the discontinuity of the chemical potential at $\nu = 1/3$. More space can be created by reversing some number of spins. For one flux quantum less than at $\nu = 1/3$, the lowest angular momentum zero energy eigenstate is the unpolarized quasielectron [6] with one reversed spin

$$\prod_{k=2}^N (z_k - z_1)^2 \prod_{m>n=2}^N (z_m - z_n)^3 |\uparrow_1 \downarrow_2 \dots \downarrow_N \rangle . \quad (4)$$

The relative angular momenta are at least 3 for pairs of electrons of the same spin and at least 2 for those of opposite spin. The total spin of the state (4) is $S = \frac{N}{2} - 1$. All zero energy states of this total spin can be obtained by multiplication of the wave function (4) with a polynomial which is symmetric in the coordinates of the spin-up electrons. In this case the polynomials are factors z_1^k with $k = 0, \dots, N - 2$. Further zero energy states can be constructed with more reversed spins, $S < \frac{N}{2} - 1$.

For two flux quanta less than at $\nu = 1/3$, a polarized ground state contains two polarized quasielectrons. In order to have zero energy, some spins must be flipped. Just one flipped spin is not enough, even if the relative angular momenta of the same spin electrons are at least 3, as in the state

$$\prod_{k=2}^N (z_k - z_1) \prod_{m>n=2}^N (z_m - z_n)^3 | \uparrow_1 \downarrow_2 \dots \downarrow_N \rangle , \quad (5)$$

where pairs of opposite spin still can have the relative angular momentum of 1. Therefore the number of reversed spins R has to be at least $R = 2$. The zero energy eigenstate of lowest angular momentum with $S = \frac{N}{2} - 2$ is

$$(z_1 - z_2)^3 \prod_{k=3}^N (z_k - z_1)^2 (z_k - z_2)^2 \prod_{m>n=3}^N (z_m - z_n)^3 | \uparrow_1 \uparrow_2 \downarrow_3 \dots \downarrow_N \rangle . \quad (6)$$

All other zero energy states of the same total spin are obtained by multiplication of Eq.(6) with polynomials symmetric in z_1, z_2 , $P_{a,b} = (z_1 + z_2)^a (z_1 - z_2)^{2b}$, such that $a + 2b \leq N - 1$. There exist further zero energy eigenstates with more reversed spins, $S < \frac{N}{2} - 2$.

In general, for Φ flux quanta less than at $\nu = 1/3$, at least Φ spins have to be reversed in order to get a zero energy eigenstate. All zero energy eigenstates of the spin $S = \frac{N}{2} - \Phi$ are obtained from a minimal angular momentum seed state

$$\prod_{i>j=1}^{\Phi} (z_i - z_j)^3 \prod_{k=\Phi+1}^N \prod_{l=1}^{\Phi} (z_k - z_l)^2 \prod_{m>n=\Phi+1}^N (z_m - z_n)^3 | \uparrow_1 \dots \uparrow_{\Phi} \downarrow_{\Phi+1} \dots \downarrow_N \rangle \quad (7)$$

by multiplication with symmetric polynomials of z_1, \dots, z_{Φ} . Further zero energy eigenstates can be constructed for $S < \frac{N}{2} - \Phi$. In the limit of $\nu = 2/5$ or $\Phi = \frac{N}{2}$, the spin singlet (3) is the unique zero energy eigenstate.

To summarize, for any filling fraction in the range from $1/3$ to $2/5$, which we characterize by the number Φ of flux quanta relative to $\nu = 1/3$, there is a degenerate band of zero energy eigenstates with spins $S = 0, \dots, \frac{N}{2} - \Phi$. The states with $S > \frac{N}{2} - \Phi$ have a nonzero interaction (2) energy. The degeneracy of the zero energy band is partially removed by arbitrarily small but nonzero Zeeman energy. Then the lowest states are those with $S = \frac{N}{2} - \Phi$. For nonzero Zeeman energy the spin of the ground state is not lower than $S = \frac{N}{2} - \Phi$. Thus for the hard core model (2) the polarization $P \equiv \frac{2S}{N}$ of the ground state at filling fraction ν is bounded from below by

$$P \geq \frac{2}{\nu} - 5 . \quad (8)$$

A similar bound holds, by particle-hole symmetry, for $\nu \in (8/5, 5/3)$. The bound is saturated for small Zeeman energy. Eq.(8) is a rigorous result for the hard core potential. In the following section we discuss its relevance for realistic Coulomb interaction.

III. SPIN POLARIZATION IN THE RANGE $1/3 < \nu \leq 2/5$ FOR COULOMB POTENTIAL

The hard core potential (2) captures important characteristics of the Coulomb potential. For instance, the states (1) and (3) are almost identical with Coulomb ground states (overlaps better than 0.99). The Coulomb potential is long ranged, however, so that its higher Haldane pseudopotentials are nonzero albeit small. This tail shifts all the energy levels and removes

the degeneracy of the hard core potential's zero energy band. As we showed in the preceding section, a nonzero pseudopotential V_1 forced a minimal number of spins to be reversed. For a Coulomb potential this tendency goes even further - the more spins are reversed the better. For Φ flux quanta less than at $\nu = 1/3$, the former band of zero energy with $S = 0, \dots, \frac{N}{2} - \Phi$ is no longer degenerate. Instead, the states with lower spin S have lower Coulomb energy. As the higher quasipotentials are weak we can expect this splitting to be small.

Let us assume that the splitting of the hard core zero energy band is small indeed. In the strong Zeeman energy limit the ground state is polarized for any filling fraction. As the Zeeman energy is decreased the spin of the ground state decreases relatively quickly from $S = \frac{N}{2}$ to $S = \frac{N}{2} - \Phi$. Within this range the scale for the gain of Coulomb energy per flipped spin is set by $V_1 - V_2$ and further gains of Coulomb energy are much smaller. If we take into account that Coulomb energy depends on the magnetic field like \sqrt{B} , further depolarization may require unrealistically small magnetic fields. We discuss the width of this split of the zero energy band of the hard core potential in two limiting cases.

A. Filling fraction just below $\nu = 2/5$

In this region the relevant degrees of freedom are quasihole excitations of the state (3)

$$\prod_{k=1}^{N/2} (z_k - w_\uparrow) \psi_{2/5}(z_k), \quad \prod_{k=(N/2)+1}^N (z_k - w_\downarrow) \psi_{2/5}(z_k) \quad (9)$$

in the spin-up and spin-down fluid respectively. N is assumed to be large. For one flux quantum more than at $\nu = 2/5$, $\Phi = \frac{N}{2} - 1$, two such quasiholes have to be created. Let us study this case in more detail.

Creation of a quasihole costs some exchange energy, which does not depend on the quasiholes's spin. If we set some definite large distance between the quasiholes, their Coulomb energy will not depend on whether they are of the same or of opposite spin. Thus the Coulomb interaction does not favor any spin polarization. It is the Zeeman energy that favors states with both quasiholes in the spin-up fluid so that the ground state's spin is $S = 1$. However, if we force the quasiholes to be closer than the magnetic length, the exchange energy will favor a pair of quasiholes in different fluids so that $S = 0$ for the Coulomb ground state.

The same logic can be applied to a finite density of quasiholes. If the average distance between quasiholes is larger than the magnetic length, states with different spin have the same Coulomb energy. Splitting may become significant only at a higher density of holes. To get an idea of how significant it is, we studied the case of one flux quantum less than at $\nu = 1/3$.

B. Filling fraction just above $\nu = 1/3$

The unpolarized quasielectron (4) with one reversed spin, $S = \frac{N}{2} - 1$, has lower Coulomb energy than its polarized counterpart, $S = \frac{N}{2}$. In order to estimate the energy gain for this depolarization we performed a finite size study of electrons in spherical geometry [4]. The

one quasiparticle sector at $\nu = 1/3$ is characterized by $3(N-1) - 1$ flux quanta piercing the sphere. We have calculated the lowest eigenstates for Coulomb interaction in the subspace of states with $S_z = \frac{N}{2} - 1$ for up to 9 electrons. The lowest energy state has always $S = N/2 - 1$, the next state $S = N/2$. The energy differences are given in the second column of the Table I. The data allow fits with quadratic polynomials in $1/N$ giving the energy gain for $N \rightarrow \infty$ of $0.0278(5)e^2/\varepsilon l$. Therefore, the unpolarized $R = 1$ quasielectron is found to be stable up to remarkably high magnetic fields $B < 22T$. Here we used parameters appropriate to GaAs ($g\mu_B B \approx 0.3K \times B[T]$, $e^2/\varepsilon l \approx 50K \times \sqrt{B[T]}$). These results are in agreement with the results in [10]. Similar calculations have been performed in the subspace $S_z = N/2 - 2$ for up to 8 electrons. The energy gains for the second depolarization are listed in the third column of the Table I. The $R = 2$ skyrmion becomes stable for $B < 0.75T$, which is remarkably small as compared to the critical magnetic field for the first depolarization.

C. Exact diagonalizations for $N = 8$ electrons

To substantiate the discussion above we have performed exact diagonalizations for $N = 8$ electrons with Coulomb interaction in spherical geometry. This gives an estimate for the splitting pattern of the zero energy band of the hard core model. Variation of the Zeeman energy induces a spectral flow. At very high Zeeman energy the ground state is fully polarized ($S = 4$). Lowering the Zeeman energy, it gives place to depolarized states with lower Coulomb energy. Table II lists the magnetic fields at which the states in the hard core zero energy band become stable. For each filling fraction (or Φ) the highest spin state $S = N/2 - \Phi$ in the band is achieved at remarkably high magnetic field but further depolarizations require much lower magnetic fields, which can not be achieved in realistic samples. This property reflects the fact that for the Coulomb interaction the pseudopotential difference $(V_1 - V_2)$ is large as compared to further differences $(V_2 - V_3)$, $(V_3 - V_4)$, which makes the Coulomb potential similar to the model hard-core interaction.

IV. UNPOLARIZED QUASIELECTRON'S SIGNATURE IN TUNNELING

Tunneling signature of the $R = 1$ skyrmion at $\nu = 1$ has been studied in the recent paper [11]. The $R = 1$ skyrmion can be interpreted as a bound state of three elementary objects: two holes in the $\nu = 1$ ferromagnet and one spin-up electron. Tunneling of either spin-up or spin-down electrons leads to characteristic features in the I-V characteristics.

The $R = 1$ unpolarized quasielectron (UQ1) can also be interpreted as a bound state of three objects: two quasiholes of charge $+e/3$ in the $\nu = 1/3$ ferromagnet and one spin-up electron of charge $-e$. This interpretation becomes clear, if we compare the UQ1 wave function (4) with the wave function for a Laughlin quasihole localized at w

$$\prod_{k=2}^N (z_k - w) \prod_{m>n=2}^N (z_m - z_n)^3 | \downarrow_2 \dots \downarrow_N \rangle . \quad (10)$$

In the wave function (4) there are two Laughlin quasiholes, both of them follow the spin-up electron, $w_1 = w_2 = z_1$. The quasiholes are in the state of relative angular momentum $L_z = -1/3$, described by the pseudo-wavefunction $(\bar{w}_1 - \bar{w}_2)^{1/3}$.

The spin-up electron constituent of the UQ1 can tunnel to another isolated $\nu = 1/3$ ferromagnet. The final state is a state with a $L_z = -1/3$ pair of quasiholes on one side of the barrier and the spin-up electron added to the $\nu = 1/3$ spin-down ferromagnet on the other side. The energy increases by the difference between the gross energy of two quasiholes and the gross energy of the UQ1. The minimal voltage V required for such a tunneling to take place is given by $eV = [2\varepsilon_-^{(1/3)} - \varepsilon_{UQ1} + E_{-1/3}] \frac{e^2}{\kappa l}$. $\varepsilon_-^{(1/3)} \approx 0.231 \frac{e^2}{\kappa l}$ according to [12]. The gross energy of the polarized quasielectron can be estimated from above [12] by $\varepsilon_+^{(1/3)} \leq -0.128 \frac{e^2}{\kappa l}$, what, together with our estimate of the depolarization energy, gives $\varepsilon_{UQ1} \leq -0.156 \frac{e^2}{\kappa l}$. $E_{-1/3}$ is the interaction energy of the two quasiholes in the $L_z = -1/3$ state, which can be estimated to be $0.040 \frac{e^2}{\kappa l}$. Therefore the tunneling of the spin-up electron occurs at the voltage $V \approx 0.66 \frac{e}{\kappa l}$.

V. CONCLUSIONS

The $R = 1$ quasielectrons above $\nu = 1/3$ (or below $\nu = 5/3$) become stable for $B < 20T$. Further depolarizations, $R > 1$, would require magnetic fields less than $0.75T$. These $R = 1$ quasielectrons dominate the depolarization pattern above $\nu = 1/3$. For low Zeeman energy the number of reversed spins is proportional to the number of quasielectrons.

We expect the following to happen in a sample as in Ref. [8]. At $\nu = 8/5$ and $B = 5T$ the ground state is a spin singlet so that $P = 0$. A decrease of the magnetic field from $B = 5T$ to $B = 4.8T$ increases the filling fraction from $\nu = 8/5$ to $\nu = 5/3$. At the same time the spin polarization, measurable with, say, Knight shift techniques, will increase from $P = 0$ to $P = 1$ according to the function $P(\nu) = \frac{2}{2-\nu} - 5$.

The unpolarized quasielectrons can manifest themselves in the I-V characteristics for tunneling between two $\nu = 1/3$ ferromagnets at $B < 20T$. In addition to the "polarized" characteristics, observable for $B > 20T$, a peak should appear at $V \approx 0.7 \frac{e}{\kappa l}$. This peak is a signature of tunneling by the spin-up electrons.

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TABLES

TABLE I. Energy difference between the lowest states of N electrons on a sphere with spin $S = N/2$ and $N/2 - 1$ at filling fraction “ $1/3 + 1$ quasiparticle”. The energy is in units of $e^2/\varepsilon l'$ with l' including a finite size correction [9] $l' = l\sqrt{\frac{m\nu}{N}}$, where m is the number of flux quanta.

N	$E_{S=N/2} - E_{S=N/2-1}$	$E_{S=N/2-1} - E_{S=N/2-2}$
5	0.04033	0.00610
6	0.03746	0.00624
7	0.03650	0.00607
8	0.03547	0.00597
9	0.03460	
∞	0.0278(5)	0.0052(1)

TABLE II. Magnetic fields at which various Coulomb energy eigenstates become stable for a model sample with $N = 8$ electrons in spherical geometry.

	$\Phi = 1$	$\Phi = 2$	$\Phi = 3$	$\Phi = 4(\nu = 2/5)$
$S = 3$	42T	—	—	—
$S = 2$	1.2T	34.4T	—	—
$S = 1$	—	0.4T	25.5T	—
$S = 0$	—	—	0.01T	17.6T